

Color van der Waals forces between heavy quarkonia in effective QCD

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The perturbative renormalization group for light-front QCD Hamiltonian produces a logarithmically rising interquark potential already in second order, when all gluons are neglected. There is a question if this approach produces also color van der Waals forces between heavy quarkonia and of what kind. This article shows that such forces do exist and estimates their strength, with the result that they are on the border of exclusion in naïve approach, while more advanced calculation is possible in QCD.

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In the constituent models used to describe hadrons, quarks can be assumed to interact via additive 2-body confining potential with the same color structure as if the interaction were mediated by one-gluon exchange,

$$H_I = - \sum_{i < j} F_i \cdot F_j V(\mathbf{r}_i - \mathbf{r}_j), \quad (1)$$

where $F_i \cdot F_j$ denotes $\sum_{a=1}^8 F_i^a F_j^a$, and F_i^a are the eight SU(3) color generators for i -th quark or antiquark, located at \mathbf{r}_i . The potential models describe properties of single-hadron states quite well, but also predict power-law color van der Waals forces between colorless hadrons, which contradict experiments by being too strong. The color confining potentials also cause that the model Hamiltonians are unbounded from below for all states except color singlets, triplets and antitriplets [?]. Potential models which try to avoid these drawbacks appear to be arbitrary and not systematically related to QCD.

One of the attempts to derive a constituent picture of hadrons directly from

QCD is by using the perturbative similarity renormalization group concept for light-front Hamiltonian [?]. ? [?], and ? [?] have shown that one can get confining potential already in second-order calculations based on this concept. In their calculation the model Hamiltonian is bounded from below for all color states of quark-antiquark pairs. However, their approach was not boost invariant and it was not clear how to describe two-meson states with clear control on their masses. Since then, the similarity renormalization group approach has advanced so that one can preserve boost invariance and required cluster properties by introducing effective particles [? , ?].

We use similarity renormalization approach (see Ref. [?] for description and references) to express the field-theoretical canonical Hamiltonian in terms of creation and annihilation operators for effective particles, which are unitarily equivalent to the canonical ones, viz. $b_\lambda = U_\lambda^\dagger b_{\text{can}} U_\lambda$ etc. The Hamiltonian H_λ written using the new operators is band-diagonal, i.e. each term is multiplied by formfactor f_λ , which vanishes when energy changes by more than width λ . We construct similarity transformation U_λ using second order perturbation theory.

Following [?], we start from the canonical QCD Hamiltonian on the light front (LF), taking the advantage of the fact that in this formulation vacuum can be made trivial by means of a cutoff on longitudinal momenta. We regulate the Hamiltonian term by term in the expansion into products of creation and annihilation operators [?], introducing regulating factors $r_{\Delta\delta}(\kappa^\perp, x) = \exp[\kappa^{\perp 2}/(x \Delta^2)] r_\delta(x)$ for each operator in every term. Here x and κ^\perp are the relative momenta of a particle with respect to other particles in a vertex (see [?]). In gauge-field theory we need r_δ , the small- x regulator, and we choose $r_\delta(x) = \Theta(x - \delta)$ as the simplest option. We use boost-invariant formfactor [?]

$$f_\lambda(\mathcal{M}_c^2 - \mathcal{M}_a^2) = \exp\left[-(\mathcal{M}_c^2 - \mathcal{M}_a^2)^2 / \lambda^4\right]. \quad (2)$$

Here $\mathcal{M}_c^2 \equiv \left(\sum_{i \in \text{creat}} p_j\right)^2$ is the free invariant mass of the particles created in the vertex, and \mathcal{M}_a^2 , similarly, for particles annihilated in vertex.

In the evaluation of H_λ , we have to calculate the effective mass m_λ for quarks and antiquarks. Following [?], we use the eigenvalue equation for a single quark state, derived in 2nd order for H_λ , at one arbitrary scale $\lambda = \lambda_0$ to specify notation for the counterterm.

$$\frac{P^{\perp 2} + \tilde{m}^2}{P^+} |q(P)\rangle = \frac{P^{\perp 2} + m_{\lambda_0}^2}{P^+} |q(P)\rangle - H_{I\lambda_0} \frac{|qg(P)\rangle \langle qg(P)|}{\mathcal{M}^2 - m^2} H_{I\lambda_0} |q(P)\rangle. \quad (3)$$

Here \mathcal{M}^2 is the free invariant mass of quark-gluon states. We express m_{λ_0} ,

presumably infinite when the cutoff δ is removed, in terms of the would-be perturbative eigenvalue \tilde{m}^2 . With hindsight, we set the eigenvalue mass \tilde{m} to be equal to the constituent quark mass. Note that this condition is not meant to imply that there are free quarks, since it is imposed only in the formal expansion in g , and ignores nonperturbative effects that prevent quarks from being free in this formulation.

We use $m_{\lambda_0}^2$ as the starting point in evaluation of m_λ^2 . Obtained quark mass $m_\lambda^2 = \tilde{m}^2 + \delta m_\lambda^2$ diverges logarithmically when δ goes to 0,

$$\delta m_\lambda^2 \xrightarrow{\delta \rightarrow 0} \frac{g^2 C_F}{2(2\pi)^3} \sqrt{\frac{\pi}{2}} \lambda^2 \log \frac{1}{\delta}. \quad (4)$$

Here C_F is the SU(3) color factor for color triplet. We would obtain the same result for δm_λ^2 in our approach if we used the coupling coherence condition [? , ? , ?].

Let us now consider the quark–antiquark interaction term in the renormalized Hamiltonian H_λ . For simplicity, we assume that all quarks have the same masses but different flavors (to avoid the insignificant discussion of antisymmetrization in the later treatment of the van der Waals forces). We will also use non relativistic (NR) approximation, which much simplifies further calculations. This approximation is justified here when studying the bound states of heavy quarks because the formfactor f_λ with $\lambda \ll \tilde{m}$ limits quark momenta to nonrelativistic values despite a large coupling constant [?]. We use a three–vector \mathbf{q}_{12} defined through the requirement that

$$4(\mathbf{q}_{12}^2 + m^2) = \mathcal{M}_{12}^2 = \frac{\kappa^{\perp 2} + m^2}{x(1-x)}, \quad \mathbf{q}_{12}^\perp = \kappa^\perp. \quad (5)$$

From this requirement one obtains x as function of q_{12}^z and \mathbf{q}_{12}^\perp , and one expands q_{12}^z in the powers of $|\mathbf{q}_{12}|/m$, obtaining in first order $q_{12}^z \simeq \left(x - \frac{1}{2}\right) 2m$. We should express the bare mass m in terms of \tilde{m} , but because the difference between m and \tilde{m} is of second order in g (see Eq. (4)), and the interaction term is already of second order, we can put $m = m_\lambda = \tilde{m}$ here.

When one applies the NR approximation to the effective potential $f_\lambda V_{q\bar{q}}$ in Hamiltonian H_λ , one obtains the following result (cf. [?])

$$f_\lambda V_{\text{NR}}(\mathbf{q}) = -g^2 F_{12} \cdot F_{34} \exp \left[-\frac{\mathbf{q}_{12}^2 - \mathbf{q}_{34}^2}{\lambda^4} \right] \times \\ \times \left\{ \frac{1}{\mathbf{q}^2} + \left[-\frac{1}{\mathbf{q}^2} + \frac{1}{q_z^2} \right] \exp \left(-\frac{2m^2 (\mathbf{q}^2)^2}{q_z^2 \lambda^4} \right) \right\} r_\delta^2 \left(\frac{q_z}{2m} \right), \quad (6)$$

where $\mathbf{q} = \mathbf{q}_{12} - \mathbf{q}_{34}$. The NR potential is spin–diagonal.

The analysis of this interaction is made somewhat complicated by the external similarity factor f_λ , which depends not only on the momentum transfer \mathbf{q} , but on $|\mathbf{q}_{12}|$ and $|\mathbf{q}_{34}|$ separately. However, when λ is much larger than momentum width of a wavefunction, f_λ can be replaced by 1 in the region that matters.

Following [? , ?], we neglect in our analysis of van der Waals forces all Fock sectors except $Q\bar{Q}$ and $QQ\bar{Q}\bar{Q}$. This is justified for heavy quarks by the expectation that gluons are lifted up in energy by certain gap condition, which is a non-perturbative effect. According to [? , ?], gluons should not appear explicitly in the resulting model.

When gluons are neglected, there remains uncanceled δ -divergent, λ -dependent term in effective quark (antiquark) mass term. This term cancels exactly in the matrix element $\langle \Psi_1 | H | \Psi_2 \rangle$ against the divergence coming from $Q\bar{Q}$ -interaction, when $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are color singlets. For other color states, the Hamiltonian matrix elements are positive infinite, since the mass diverges stronger than the potential of Eq. (6) [?]. Therefore, the Hamiltonian is bounded from below for all states (as opposed to phenomenological additive color potential model where $\langle \Psi_1 | H | \Psi_2 \rangle$ goes to $-\infty$ with growing distance between quarks when both $|\Psi_i\rangle$ are in color-octet state, for all $V(r) \xrightarrow{r \rightarrow \infty} \infty$ [?]). This is the first important distinction between the renormalization group approach to QCD and potential models, where the colored states would cause trouble.

Note however that this cancellation does not occur outside the NR approximation [?]. One has to include transitions to $q\bar{q}g$ sector to obtain cancellation outside NR approximation. The second order perturbation calculations would then also lead to reduction of the confining part of effective potential, but non-perturbative effects could prevent complete cancellation [?], resulting only in weaker force.

Color van der Waals forces between two mesons come from the mixing of octet-octet state (coupled to overall singlet) to singlet-singlet state, because the color dependent interaction polarizes mesons in color space as well as in position space. We choose states $|\alpha\rangle = |1_{13}1_{24}\rangle$ (both mesons in singlet state) and $|\beta\rangle = |1_{14}1_{23}\rangle$, as the basis of the color subspace, where mesons are in color singlet state as a whole, where 1_{ij} means that quark i and antiquark j are in color singlet.

We will use static treatment to see if small- x divergences (which are constants) cancel in the eigenequation for $QQ\bar{Q}\bar{Q}$ sector. For the meson-meson states, the interaction term H_I is a 2×2 matrix in color space [?]:

$$H_I|\alpha\rangle = (\frac{8}{3}u_\alpha - \frac{1}{3}u_\beta + \frac{1}{3}u_q)|\alpha\rangle + (u_\beta - u_q)|\beta\rangle, \quad (7a)$$

$$H_I|\beta\rangle = (u_\alpha - u_q)|\alpha\rangle + (\frac{8}{3}u_\beta - \frac{1}{3}u_\alpha + \frac{1}{3}u_q)|\beta\rangle, \quad (7b)$$

where

$$u_\alpha = u_{13} + u_{24}, \quad u_\beta = u_{14} + u_{23}, \quad u_q = u_{12} + u_{34}, \quad (8)$$

and $u_{ij} = V(\mathbf{r}_{ij}) = V(\mathbf{r}_i - \mathbf{r}_j)$ is the quark-(anti)quark potential. The mass term is diagonal in $|\alpha\rangle, |\beta\rangle$. The divergences in the off diagonal (mixing) part cancel out because $V_{\bar{q}q} = V_{qq} = -V_{q\bar{q}}$. In the diagonal part, divergences in masses cancel with divergences in potential inside singlets (i.e. u_α and u_β , respectively) and divergences in the $u_q - u_\beta$ and $u_q - u_\alpha$ cancel also. Thus, all small- x divergences cancel out and Eq. (7) has finite elements. Therefore, color van der Waals forces emerge in the approach presented here as they appear in color potential models. This is the second result of the effective particle approach in the approximation proposed in [? , ?]. The third result is that we may then use the results on color van der Waals forces as known from potential models.

To see what the van der Waals force looks like we have to look at the NR potential (6) behavior at large distances. We will use the spatial derivatives of the confining potential [?]. The partial derivative with respect to r_x for $r_z = r_y = 0$, \mathbf{r} being the distance between Q and \bar{Q} in quarkonium state, is given by the equation

$$\frac{\partial}{\partial r_x} V_{\text{conf}}(r_x) \simeq \frac{\alpha_s \lambda^2}{2\pi m} 2\sqrt{2\pi} \frac{\partial}{\partial r_x} \log|r_x/r_0| + \text{short range} \quad (9)$$

(the result for the r_y direction is identical). Similar calculations of the shape of the potential in the r_z direction give in the large distance limit the same functional form of the potential but twice weaker. Equation (9) contains unknown value of r_0 , but from the fit to numerically obtained Fourier transformation one gets r_0 of the order of m/λ^2 .

To simplify further calculations, we will use V_{conf} averaged over angles (like in [?]). Numerical calculations show that with growth of r the range of polar angle Θ , where V_{conf} differs significantly from the value for $\Theta = \pi/2$, gets smaller and smaller. Therefore, for the large r we can use the analytical result for the \mathbf{r}_\perp direction (9), getting the potential of the form

$$u(r) = V_0 \log(r/r_0), \quad \text{where } V_0 = 2\alpha_s \lambda^2 / [\sqrt{2\pi} m]. \quad (10)$$

For such logarithmic potential one obtains from the eigenvalues of 2×2 matrix in Eq. (7) and virial theorem [?] the following color van der Waals force between two heavy mesons:

$$V_{\text{vdW}}(R) = \frac{3}{54} V_0 \frac{\langle r_{13}^2 \rangle^2}{R^4} \frac{1}{\log(R/r_{13})}, \quad (11)$$

where R is the distance between the mesons and r_{13} is the mean width of a meson. Note that the value of r_0 does not enter the expression for the van der Waals force (11).

The fit to the B meson spectra via heavy quark effective theory in [?] and the fit to the $c\bar{c}$ and $b\bar{c}$ spectra in [?] give $\alpha_s \lambda^2/m$ around 1 GeV, i.e. $V_0 \simeq 0.8$ GeV. In color potential model (1) with $V(r) = V_0 \log(r/r_0)$, one obtains from charmonium data $V_0 = 731$ GeV [?, ?]. The question is then if such well established value of V_0 produces acceptable strength of color van der Waals potential (11).

We have done our calculations of the effective confining potential in the NR limit, which strictly speaking can be valid only for heavy quarkonia. To the author's best knowledge there currently exist no data on forces between such mesons. Therefore, we will use existing experimental limits on the color van der Waals forces between *light hadrons*, to make only order of magnitude guesses about the allowed strength of V_0 .

Between two (heavy) baryons one obtains the same type of force as between two (heavy) mesons but with different coefficient (e.g. [?] gives 1/2 for baryons instead of 3/54 for mesons, but they use a different method to calculate van der Waals force coming from potential (10)). Our NR potential (6) does not apply to light quarks, but because van der Waals force is a low-energy interaction acting between slow-moving particles, and in constituent models NR approximation is commonly used, we will treat the NR equation (11) for the van der Waals potential as providing approximately right order of magnitude, disregarding any doubts that such procedure may no longer be adequate in the case of effective QCD for light quarks, where precise comparison with data is attempted.

Data from Cavendish-type experiments [?] give no real constraints on the van der Waals force resulting from logarithmic inter-quark potential, as opposed to the one from linear potential [?]. The data from hadronic atoms [?] implies $(3/54)V_0 \lesssim 2$ MeV i.e. $V_0 \lesssim 40$ MeV for $r_{13} = 1$ fm. We see that $V_0 \simeq 0.8$ GeV appears to be very large. [?, ?, ?] finds in the behavior of P-wave amplitude of π - π and S-wave p-p scattering at low energy some signs of long range super-strong force $\propto 1/R^n$ with n about 6, but no trace of forces $\propto 1/[R^4 \log(R)]$ is found. While the strength of his force is too large to be explained using α_s that appears in Eq. (10), and remains to be explained, it is interesting that van der Waals force $\propto 1/R^6$ comes from Coulombic part of the inter-quark potential, which cannot be easily canceled.

It is clear that the calculation reported here can be considered only an initial step towards derivation of color van der Waals forces in QCD. The non-cancellation of small- x divergences without NR approximation, and the lack of rotational symmetry of $V_{q\bar{q}}$, suggest that one needs to consider Fock sectors

with additional gluons. Most prominently however, one should expect corrections from creation of quark–antiquark pairs [?]. These elements can reduce the van der Waals forces we obtained here and recover agreement with experimental findings. The most attractive feature of presented approach is that, contrary to the potential models, the effective particle picture is clearly open to further study in QCD, and can incorporate all these effects starting from first principles.